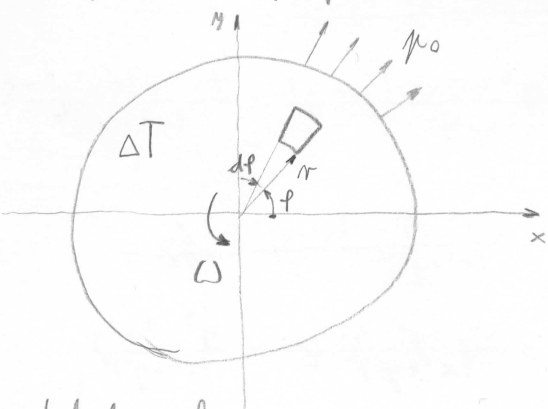
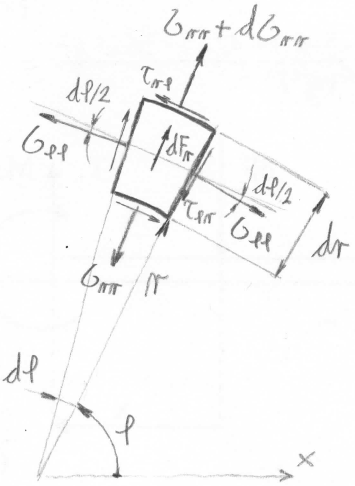
Obravnavaj rotirajoče, osnosimetrično obremenjene krožne stene v konstantnem temperaturnem polju. Predpostavite RNS. Krožna stena je obremenjena v elastičnem območju.



debelina: h



RNS:

$$\sigma_{zz} = 0$$

$$\tau_{zr} = 0 \qquad \tau_{z\varphi} = 0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\tau_{rz} = 0 \qquad \tau_{\varphi z} = 0$$

osna simetrija: $\frac{\partial}{\partial \varphi} = 0$; $\tau_{\varphi r} = 0 \Rightarrow \tau_{r\varphi} = 0$

centripetalna sila dF_r : $dF_r = dm \cdot a_r = \rho \cdot dV \cdot r \cdot \omega^2 = \rho \cdot r \cdot d\varphi \cdot dr \cdot h \cdot r \cdot \omega^2$

$$dF_r = \rho r^2 \cdot \omega^2 \cdot h \cdot dr \cdot d\varphi$$

$$\sum_i F_{ir} = 0 = (\sigma_{rr} + d\sigma_{rr})(r + dr) \cdot d\varphi \cdot h - \sigma_{rr} \cdot r \cdot d\varphi \cdot h - \sigma_{\varphi\varphi} \sin\frac{d\varphi}{2} \cdot dr \cdot h \cdot 2 + dF_r$$

$$\cancel{\sigma_{rr} r\, d\varphi\, h} + \sigma_{rr}\, dr\, d\varphi \cdot h + d\sigma_{rr}\, r\, d\varphi\, h + d\sigma_{rr}\, dr\, d\varphi \cdot h - \cancel{\sigma_{rr} r\, d\varphi\, h} - \sigma_{\varphi\varphi}\, d\varphi \cdot dr \cdot h + \rho r^2 \omega^2 h \cdot dr \cdot d\varphi = 0 \quad \Big\} \cdot \frac{1}{d\varphi \cdot h}$$

$$\sigma_{rr}\, dr + d\sigma_{rr}\, r + \underbrace{d\sigma_{rr} \cdot dr}_{\sim 0} - \sigma_{\varphi\varphi}\, dr + \rho r^2 \omega^2 \cdot dr = 0 \quad \Big\} \cdot \frac{1}{dr}$$

$$\sigma_{rr} + r \cdot \frac{d\sigma_{rr}}{dr} - \sigma_{\varphi\varphi} + \rho r^2 \omega^2 = 0 \Rightarrow \boxed{\sigma_{\varphi\varphi} = r \cdot \frac{d\sigma_{rr}}{dr} + \sigma_{rr} + \rho \omega^2 r^2}$$

$$\sum_i F_{i\varphi} = 0 = \sigma_{\varphi\varphi} \cos\frac{d\varphi}{2}\, dr \cdot h - \sigma_{\varphi\varphi} \cdot \cos\frac{d\varphi}{2}\, dr \cdot h \Rightarrow \quad 0 = 0 \;\checkmark$$

$$\sum_i F_{iz} = 0 = 0 \;\checkmark$$

HOOKE-ov zakon:

$$\boxed{\begin{aligned} \varepsilon_{rr} &= \frac{1}{E}\left[\sigma_{rr} - \nu \cdot (\sigma_{\varphi\varphi} + \cancelto{0}{\sigma_{zz}})\right] + \alpha \cdot \Delta T \\ \varepsilon_{\varphi\varphi} &= \frac{1}{E}\left[\sigma_{\varphi\varphi} - \nu \cdot (\cancelto{0}{\sigma_{zz}} + \sigma_{rr})\right] + \alpha \cdot \Delta T \\ \varepsilon_{zz} &= \frac{1}{E}\left[\cancelto{0}{\sigma_{zz}} - \nu \cdot (\sigma_{rr} + \sigma_{\varphi\varphi})\right] + \alpha \cdot \Delta T \end{aligned}}$$

ker so vse strižne napetosti enake nič: $\tau_{r\varphi} = \tau_{\varphi z} = \tau_{zr} = 0$ so tudi vse strižne deformacije enake nič: $\varepsilon_{r\varphi} = \varepsilon_{\varphi z} = \varepsilon_{zr} = 0$ $\quad$ ($\varepsilon_{r\varphi} = \frac{1}{2G} \cdot \tau_{r\varphi}$ itd....)

potrebujemo še eno enačbo in to je kompatibilnostna enačba, ki je v primeru osne simetrije v cilindričnih koordinatah enaka:

$$\varepsilon_{rr} = r \frac{d\varepsilon_{\varphi\varphi}}{dr} + \varepsilon_{\varphi\varphi}$$

Zdaj imamo pet enačb in pet neznank ($\sigma_{rr}, \sigma_{\varphi\varphi}, \varepsilon_{rr}, \varepsilon_{\varphi\varphi}, \varepsilon_{zz}$) in problem je možno rešiti:

$$r \cdot \frac{d\varepsilon_{\varphi\varphi}}{dr} + \varepsilon_{\varphi\varphi} = \frac{1}{E}(\sigma_{rr} - \nu \cdot \sigma_{\varphi\varphi}) + \alpha \cdot \Delta T$$

$$r \cdot \frac{1}{E}\left(\frac{d\sigma_{\varphi\varphi}}{dr} - \nu \cdot \frac{d\sigma_{rr}}{dr}\right) + \frac{1}{E}(\sigma_{\varphi\varphi} - \nu \cdot \sigma_{rr}) + \alpha \cdot \Delta T = \frac{1}{E}(\sigma_{rr} - \nu \sigma_{\varphi\varphi}) + \alpha \cdot \Delta T$$

vstavimo še ravnotežno enačbo:

$$\frac{d\sigma_{\varphi\varphi}}{dr} = \frac{d\sigma_{rr}}{dr} + r \frac{d^2\sigma_{rr}}{dr^2} + \frac{d\sigma_{rr}}{dr} + 2\rho\omega^2 r$$

$$\frac{d\sigma_{\varphi\varphi}}{dr} = r \cdot \frac{d^2\sigma_{rr}}{dr^2} + 2 \cdot \frac{d\sigma_{rr}}{dr} + 2\rho\omega^2 r$$

in na koncu dobimo:

$$r^2 \cdot \frac{d^2\sigma_{rr}}{dr^2} + 3r \frac{d\sigma_{rr}}{dr} = -(3+\nu)\rho\omega^2 r^2$$

EULER-jeva nehomogena $\overset{\text{LINEARNA}}{\text{diferencialna}}$ enačba 2. reda z nekonstantnimi koeficienti

upoštevamo: $r = e^t$

in na koncu dobimo:

$$\sigma_{rr} = A + Br^{-2} - \frac{3+\nu}{8}\rho\omega^2 r^2$$

in potem iz ravnotežne enačbe:

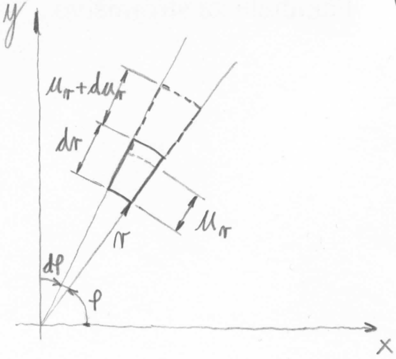
$$\sigma_{\varphi\varphi} = A - Br^{-2} - \frac{1+3\nu}{8}\rho\omega^2 r^2$$

specifične deformacije ε_{rr}, $\varepsilon_{\varphi\varphi}$ in ε_{zz} lahko brez težav določimo iz HOOKE-ovega zakona. Običajno nas bolj zanimajo premiki: $(\vec{u}) = (u_r, u_\varphi, u_z)$

zaradi osne simetrije, mora biti $u_\varphi = 0$

$\Rightarrow (\vec{u}) = (u_r, 0, u_z)$

najbolj nas zanima premik u_r.



$$(\vec{u})=(u_r, 0, u_z)$$

$$\varepsilon_{rr}=\frac{L_{rk}-L_{r0}}{L_{r0}}=\frac{(u_r+du_r+dr-u_r)-dr}{dr}=\frac{du_r}{dr}$$

$$\varepsilon_{\varphi\varphi}=\frac{L_{\varphi k}-L_{\varphi 0}}{L_{\varphi 0}}=\frac{(r+u_r)\cdot d\varphi-r\cdot d\varphi}{r\cdot d\varphi}=\frac{u_r}{r}$$

$$\varepsilon_{zz}=\frac{du_z}{dz} \Rightarrow u_z=\int\varepsilon_{zz}\,dz$$

$$u_r=\varepsilon_{\varphi\varphi}\cdot r$$

$$u_r=\frac{1}{E}\left[(1-\nu)\cdot A\cdot r-\frac{(1+\nu)}{r}\cdot B-\frac{1-\nu^2}{8}\rho\omega^2 r^3\right]+\alpha\cdot\Delta T\cdot r$$

$$u_z=\frac{\nu\cdot z}{E}\left[\frac{1+\nu}{2}\cdot\rho\omega^2 r^2-2\cdot A\right]+\alpha\cdot\Delta T\cdot z$$

RNS

$$u_\varphi=0$$

$$\sigma_{rr}=A+B\cdot r^{-2}-\frac{3+\nu}{8}\rho\omega^2 r^2 \qquad \sigma_{zz}=0 \quad ; \quad u_\varphi=0$$

$$\sigma_{\varphi\varphi}=A-B\cdot r^{-2}-\frac{1+3\nu}{8}\rho\omega^2 r^2$$

$$(\sigma_{ij})=\begin{pmatrix}\sigma_{rr}&0&0\\0&\sigma_{\varphi\varphi}&0\\0&0&0\end{pmatrix} \qquad (\varepsilon_{ij})=\begin{pmatrix}\varepsilon_{rr}&0&0\\0&\varepsilon_{\varphi\varphi}&0\\0&0&\varepsilon_{zz}\end{pmatrix} \qquad \vec{u}=(u_r, 0, u_z)$$

v primeru RDS imamo spet pet enačb in pet neznank $(\sigma_{rr}, \sigma_{\varphi\varphi}, \sigma_{zz}, \varepsilon_{rr}, \varepsilon_{\varphi\varphi})$

$\varepsilon_{zz}=0$; strižne napetosti in strižne specifične deformacije so spet enake 0.

$\Downarrow$ $u_z=0$; zaradi simetrije je u_φ spet enak 0.

$$\sigma_{rr}=A+Br^{-2}-\frac{3-2\nu}{8(1-\nu)}\rho\omega^2 r^2$$

$$\sigma_{\varphi\varphi}=A-Br^{-2}-\frac{1+2\nu}{8(1-\nu)}\rho\omega^2 r^2$$

RDS

$$\sigma_{zz}=2\nu\cdot A-\frac{\nu}{2(1-\nu)}\rho\omega^2 r^2-\alpha\cdot\Delta T\cdot E$$

$$u_r=\frac{1+\nu}{E}\left[(1-2\nu)\cdot Ar-\frac{B}{r}-\frac{1-2\nu}{8(1-\nu)}\rho\omega^2 r^3\right]+(1+\nu)\cdot\alpha\cdot\Delta T\cdot r$$

$$\varepsilon_{zz}=0 \quad u_z=0 \quad u_\varphi=0$$

$$(\sigma_{ij})=\begin{pmatrix}\sigma_{rr}&0&0\\0&\sigma_{\varphi\varphi}&0\\0&0&\sigma_{zz}\end{pmatrix} \qquad (\varepsilon_{ij})=\begin{pmatrix}\varepsilon_{rr}&0&0\\0&\varepsilon_{\varphi\varphi}&0\\0&0&0\end{pmatrix} \qquad \vec{u}=(u_r, 0, 0)$$