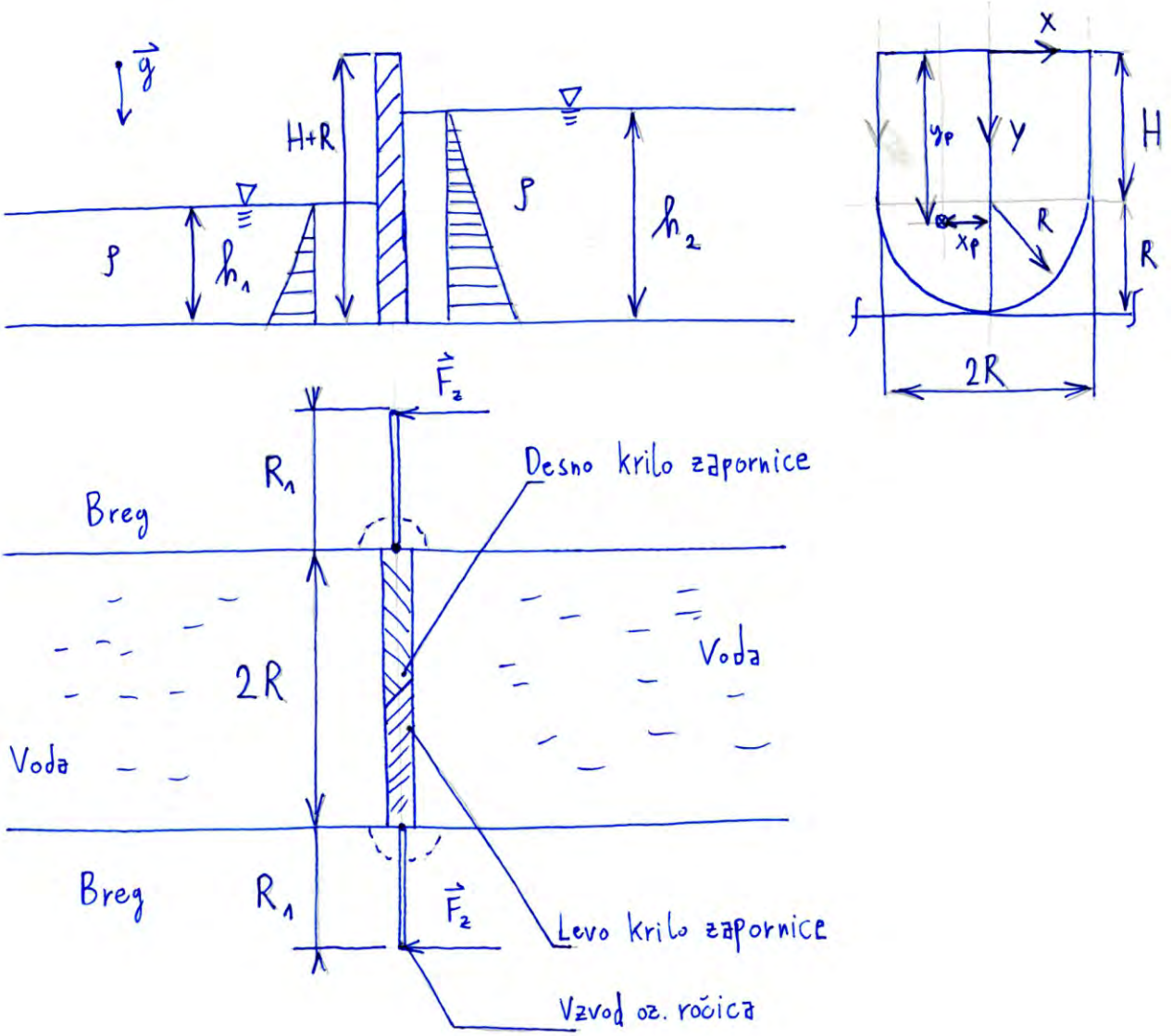
Naloga 2. Zapornica, ki zapira prekat na kanalu, je sestavljena iz dveh kril. Spoj med kriloma je zatesnjen. Kolikšni morata biti sili na ročici oz. vzvoda (Fz), če prek njih držimo zapornico zaprto? Določi tudi položaja prijemališča sile na posamezno krilo zapornice (upoštevaj koordinatni sistem, definiran na risbi).



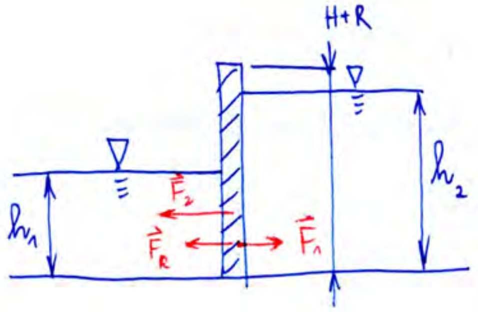
Podatki:

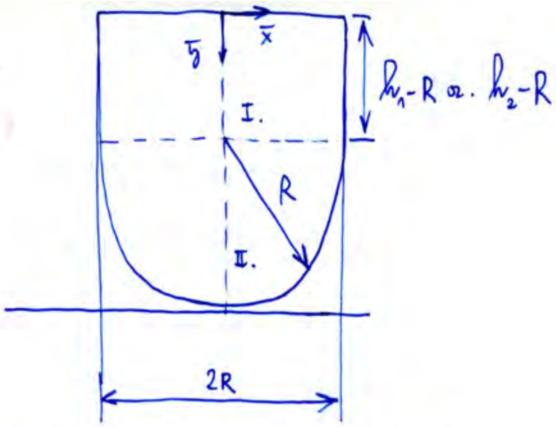
$\rho = 1000 \frac{kg}{m^3}$ $h_{v_1} = 2{,}5\,m$ $H = 3\,m$ $R_1 = 2\,m$

$g = 9{,}807 \frac{m}{s^2}$ $h_{v_2} = 3{,}8\,m$ $R = 1{,}5\,m$

$F_z = ?$, $x_p = ?$, $y_p = ?$

(x_p, y_p ... enakovredno x_F, y_F - prijemališče sile)





↳ Tako za levo kot desno stran (glede na kanal privzamemo, da se koord. sist. $\bar{x}$-$\bar{y}$ nahaja na gladini)

→ Leva stran:

$$A_1 = 2R(h_1-R)+\frac{\pi R^2}{2} = 2\cdot 1{,}5\,m\cdot(2{,}5\,m-1{,}5\,m)+\frac{\pi\cdot(2{,}5\,m)^2}{2} = \sout{\ldots} = \underline{6{,}5343\,m^2}$$

$$\bar{y}_{T_1} = \frac{2R(h_1-R)\cdot\frac{1}{2}(h_1-R)+\frac{\pi R^2}{2}\left((h_1-R)+\frac{4R}{3\pi}\right)}{2R(h_1-R)+\frac{\pi R^2}{2}} = \frac{2\cdot 1{,}5\,m\cdot\frac{1}{2}\cdot(1\,m)^2+\frac{\pi\cdot 2\,m(1{,}5\,m)^2}{2}\left(1\,m+\frac{4\cdot 1{,}5\,m}{3\pi}\right)}{2\cdot 1{,}5\,m\cdot 1\,m+\frac{\pi(1{,}5\,m)^2}{4}} =$$

$$= \frac{7{,}2843\,m^3}{6{,}5343\,m^2} = \underline{1{,}115\,m}$$

A_2

→ Desna stran:

$$A_2 = 2R(h_2-R)+\frac{\pi R^2}{2} = 2\cdot 1{,}5\,m\cdot(3{,}8\,m-1{,}5\,m)+\frac{\pi(1{,}5\,m)^2}{2} = \underline{10{,}4343\,m^2}$$

$$\bar{y}_{T_2} = \frac{2R(h_2-R)\cdot\frac{1}{2}(h_2-R)+\frac{\pi R^2}{2}\left((h_2-R)+\frac{4R}{3\pi}\right)}{2R(h_2-R)+\frac{\pi R^2}{2}} = \frac{2\cdot 1{,}5\,m\cdot\frac{1}{2}(3{,}8\,m-1{,}5\,m)^2+\frac{\pi(1{,}5\,m)^2}{2}\left(2{,}3\,m+\frac{4\cdot 1{,}5\,m}{3\pi}\right)}{2\cdot 1{,}5\,m\,(3{,}8\,m-1{,}5\,m)+\frac{\pi(1{,}5\,m)^2}{2}} =$$

$$= \frac{18{,}3139\,m^3}{10{,}4343\,m^2} = \underline{1{,}738\,m}$$

→ Rez. sile:

$$F_R = \rho g(\bar{y}_{T_2}A_2-\bar{y}_{T_1}A_1) = 1000\,\frac{kg}{m^3}\cdot 9{,}807\,\frac{m}{s^2}\cdot\left(1{,}738\,m\cdot 10{,}4343\,m^2-1{,}115\,m\cdot 6{,}5343\,m^2\right) =$$

$$= \underline{106{,}397\,kN}$$

→ Vsota momentov: $R_1F_z - RF_R = 0$

$$F_z = \frac{R}{R_1}F_R = \frac{1{,}5\,m}{2\,m}\cdot 106{,}397\,kN = \boxed{79{,}798\,kN}$$

→ Prijemališče sile:

↳ Ker je razporeja simetrična glede na y os, je koordinata prijemališča enaka za obe krili vrat, x koordinati pa imata nasprotni predznak.

$$\bar{y}_F = \bar{y}_T + \frac{I_{\bar{x}_T}}{\bar{y}_T A_I}, \quad \bar{x}_F = \bar{x}_T + \frac{I_{\bar{x}\bar{y}_T}}{\bar{y}_T A_I}$$

→ Leva stran (h_1):

$$\bar{x}_T = \frac{\bar{x}_{T,I} A_I + \bar{x}_{T,II} A_{II}}{A_I + A_{II}} = \frac{\frac{1}{2} R \cdot R(h_1 - R) + \frac{4R}{3\pi} \cdot \frac{\pi R^2}{4}}{\frac{1}{2}\left(2R(h_1-R) + \frac{\pi R^2}{2}\right)} =$$

$$= \frac{\frac{1}{2}(1{,}5m)^2 \cdot 1m + \frac{(1{,}5m)^3}{3}}{\frac{1}{2} \cdot 6{,}5343 m^3} = \frac{2{,}25 m^3}{3{,}2671 m^2} = \underline{0{,}6887 m}$$

$$\bar{y}_T = \frac{\bar{y}_{T,I} A_I + \bar{y}_{T,II} A_{II}}{A_I + A_{II}} = \frac{\frac{1}{2}(h_1 - R) \cdot R(h_1 - R) + \left(\left(h_1 - R + \frac{4R}{3\pi}\right) \cdot \frac{\pi R^2}{4}\right)}{\frac{1}{2}\left(2R(h_1 - R) + \frac{\pi R^2}{2}\right)} =$$

$$= \frac{\frac{1}{2}(1m)^2 \cdot 1{,}5m + \left(\left(1m + \frac{4 \cdot 1{,}5m}{3\pi}\right) \frac{\pi (1{,}5m)^2}{4}\right)}{3{,}2671 m^2} =$$

$$= \frac{0{,}75 m^3 + 2{,}8921 m^3}{3{,}2671 m^2} = \underline{1{,}1148 m}$$

$$I_{\bar{x}_{T,I}} = \frac{R(h_1-R)^3}{12} = \frac{1{,}5m (1m)^3}{12} = 0{,}125 m^4, \quad I_{\bar{x}_{T,II}} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) R^4 = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)(1{,}5m)^4 = 0{,}2778 m^4$$

$$I_{\bar{x}\bar{y}_{T,I}} = 0 m^4, \quad I_{\bar{x}\bar{y}_{T,II}} = \frac{9\pi - 32}{72\pi} R^4 = -0{,}0834 m^4$$

| $y_{T,I} = \frac{1}{2}(h_1 - R)$; $x_{T,I} = \frac{1}{2} R$
| $y_{T,II} = (h_1 - R) + \frac{4R}{3\pi}$, $x_{T,II} = \frac{4R}{3\pi}$
| $A_I = (h_1 - R)R$, $A_{II} = \frac{1}{4}\pi R^2$

$$I_{\bar{x}_T} = I_{\bar{x}_{T,I}} + (\bar{y}_{T,I} - \bar{y}_T)^2 A_I + I_{\bar{x}_{T,II}} + (\bar{y}_{T,II} - \bar{y}_T)^2 A_{II} =$$

$$= 0{,}125 m^4 + (0{,}5m - 1{,}1148m)^2 \cdot 1{,}5 m^2 + 0{,}2778 m^4 + (1{,}6366m - 1{,}1148m)^2 \cdot 1{,}7671 m^2 = \underline{1{,}4509 m^4}$$

$$I_{\bar{x}\bar{y}_T} = I_{\bar{x}\bar{y}_{T,I}} + (\bar{y}_{T,I} - \bar{y}_T)(\bar{x}_{T,I} - \bar{x}_T) A_I + I_{\bar{x}\bar{y}_{T,II}} + (\bar{x}_{T,II} - \bar{x}_T)(\bar{y}_{T,II} - \bar{y}_T) A_{II} =$$

$$= 0 m^4 + (0{,}5m - 1{,}1148m)(0{,}75m - 0{,}6887m) \cdot 1{,}5 m^2 - 0{,}0834 m^4 + (0{,}6366m - 0{,}6887m)(1{,}6366m - 1{,}1148m) \cdot 1{,}7671 m^2 = \underline{-0{,}188 m^4}$$

$$x_F = 0{,}6887\,m + \frac{-0{,}188\,m^4}{1{,}1148\,m \cdot 3{,}2671\,m^2} = \underline{0{,}6371\,m}\,, \quad y_F = 1{,}1148\,m + \frac{1{,}4509\,m^4}{1{,}1148\,m \cdot 3{,}2671\,m^2} = \underline{1{,}5132\,m}$$

↳ V glavnem koordinatnem sistemu:

1. Prijemališče: $\boxed{x_{F_1} = -0{,}6371\,m\,,\ y_{F_1} = 3{,}5132\,m}$
2. Prijemališče: $\boxed{x_{F_2} = 0{,}6371\,m\,,\ y_{F_2} = 3{,}5132\,m}$

→ Desna stran (h_2):

$$\bar{x}_T = \frac{\bar{x}_{T,I} A_I + \bar{x}_{T,II} A_{II}}{A_I + A_{II}} = \frac{\frac{1}{2} R \cdot (h_2 - R) R + \frac{4R}{3\pi} \cdot \frac{\pi R^2}{4}}{\frac{1}{2}\left(2R(h_2 - R) + \frac{\pi R^2}{2}\right)} =$$

$$= \frac{\frac{1}{2}(1{,}5\,m)^2 \cdot 2{,}3\,m + \frac{(1{,}5\,m)^3}{3}}{\frac{1}{2} \cdot 10{,}4343\,m^2} = \frac{3{,}7125\,m^3}{5{,}2772\,m^2} = \underline{0{,}7035\,m}$$

$$\bar{y}_T = \frac{\bar{y}_{T,I} A_I + \bar{y}_{T,II} A_{II}}{A_I + A_{II}} = \frac{\frac{1}{2}(h_2 - R) \cdot R(h_2 - R) + \left((h_2 - R) + \frac{4R}{3\pi}\right) \cdot \frac{\pi R^2}{4}}{\frac{1}{2}\left(2R(h_2 - R) + \frac{\pi R^2}{2}\right)} =$$

$$= \frac{\frac{1}{2} \cdot 1{,}5\,m \cdot (2{,}3\,m)^2 + \left(2{,}3\,m + \frac{4 \cdot 1{,}5\,m}{3\pi}\right) \cdot \frac{1}{4}\pi (1{,}5\,m)^2}{5{,}2772\,m^2} = \frac{3{,}9675\,m^3 + 5{,}1894\,m^3}{5{,}2772\,m^2} = \underline{1{,}7352\,m}$$

$$I_{\bar{x}_{T,I}} = \frac{1}{12} R (h_2 - R)^3 = \frac{1}{12} 1{,}5\,m \cdot (2{,}3\,m)^3 = 1{,}5209\,m^4\,, \quad I_{x_{T,II}} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) R^4 = 0{,}2778\,m^4$$

$$I_{\bar{x}\bar{y}_{T,I}} = 0\,m^4\,, \quad I_{\overline{xy}_{T,II}} = \frac{(9\pi - 32) R^4}{72\pi} = -0{,}0834\,m^4$$

$$I_{\bar{x}_T} = I_{\bar{x}_{T,I}} + (\bar{y}_{T,I} - \bar{y}_T)^2 A_I + I_{\bar{x}_{T,II}} + (\bar{y}_{T,II} - \bar{y}_T)^2 A_{II} =$$

$$= 1{,}5209\,m^4 + (1{,}15\,m - 1{,}7352\,m)^2 \cdot 3{,}45\,m^2 + 0{,}2778\,m^4 +$$

$$+ (2{,}9366\,m - 1{,}7352\,m)^2 \cdot 1{,}7671\,m^2 = \underline{5{,}5307\,m^4}$$

$y_{T,I} = \frac{1}{2}(h_2 - R)$; $x_{T,I} = \frac{1}{2} R$

$y_{T,II} = (h_2 - R) + \frac{4R}{3\pi}$; $x_{T,II} = \frac{4R}{3\pi}$

$A_I = (h_2 - R) R$, $A_{II} = \frac{1}{4}\pi R^2$

$$I_{\bar{x}\bar{y}_T} = I_{\bar{x}\bar{y}_{T,I}} + (\bar{x}_{T,I} - \bar{x}_T)(\bar{y}_{T,I} - \bar{y}_T) A_I + I_{\bar{x}\bar{y}_{T,II}} + (\bar{x}_{T,II} - \bar{x}_T)(\bar{y}_{T,II} - \bar{y}_T) A_{II} =$$

$$= 0\,m^4 + (0{,}75\,m - 0{,}7035\,m)(1{,}15\,m - 1{,}7352\,m) \cdot 3{,}45\,m^2 - 0{,}0834\,m^4 +$$

$$+ (0{,}6366\,m - 0{,}7035\,m)(2{,}9366\,m - 1{,}7352\,m) \cdot 1{,}7671\,m^2 = \underline{-0{,}3193\,m}$$

$$x_F = 0{,}7035\,m + \frac{-0{,}3193\,m^4}{1{,}7352\,m \cdot 5{,}2772\,m^2} = \underline{0{,}6886\,m}$$

$$y_F = 1{,}7352\,m + \frac{5{,}5307\,m^4}{1{,}7352\,m \cdot 5{,}2772\,m^2} = \underline{2{,}3392\,m}$$

↳ V glavnem koordinatnem sistemu:

1. Prijemališče: $x_{F_1} = -0{,}6886\,m\,,\ y_{F_1} = 3{,}0352\,m$
2. Prijemališče $x_{F_2} = 0{,}6886\,m\,,\ y_{F_2} = 3{,}0352\,m$