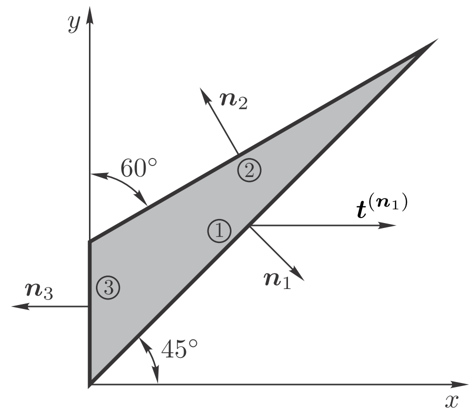
Naloga. Za narisani primer določite napetostna vektorja $\boldsymbol{t}^{(\boldsymbol{n}_2)}$ in $\boldsymbol{t}^{(\boldsymbol{n}_3)}$, da bo izpolnjeno statično ravnotežje. Določite tudi komponente tenzorja napetosti σ_{ij}, velikosti ekstremnih normalnih napetosti σ_1, σ_2, njihovo lego (α), velikost ekstremnih strižnih napetosti $\sigma_{t,\max}$, njihovo lego (β) ter velikost normalnih napetosti, ki se pojavijo na ravninah z ekstremnimi strižnimi napetostmi, če je $(\boldsymbol{t}^{(\boldsymbol{n}_1)}) = (60.0, 0)$ MPa in $\sigma_{xx} = -30.0$ MPa. Predpostavite homogeno ravninsko napetostno stanje.



Iz zgornje slike lahko zapišemo

$$
\begin{aligned}
(\boldsymbol{n}_1) &= (\cos 45^\circ, -\sin 45^\circ)\\
(\boldsymbol{n}_2) &= (-\cos 60^\circ, \sin 60^\circ)\\
(\boldsymbol{n}_3) &= (-1.0, 0).
\end{aligned}
$$

Komponente tenzorja napetosti določimo s pomočjo izraza $t_i^{(\boldsymbol{n})} = \sigma_{ij} n_j$, ki ga uporabino na prvi ravnini,

$$
\begin{aligned}
t_x^{(\boldsymbol{n}_1)} &= \sigma_{xx} n_{1x} + \sigma_{xy} n_{1y}\\
60.0 &= -30.0 \cos 45^\circ + \sigma_{xy}(-\sin 45^\circ)\\
\Rightarrow &\quad \sigma_{xy} = -114.853 \text{ MPa},
\end{aligned}
$$

$$
\begin{aligned}
t_y^{(\boldsymbol{n}_1)} &= \sigma_{xy} n_{1x} + \sigma_y n_{1y}\\
0 &= -114.853 \cos 45^\circ + \sigma_{yy}(-\sin 45^\circ)\\
\Rightarrow &\quad \sigma_{yy} = -114.853 \text{ MPa},
\end{aligned}
$$

pri čemer je $(\boldsymbol{t}^{(\boldsymbol{n}_1)}) = (t_x^{(\boldsymbol{n}_1)}, t_y^{(\boldsymbol{n}_1)})$ in $(\boldsymbol{n}_1) = (n_{1x}, n_{1y})$. Torej

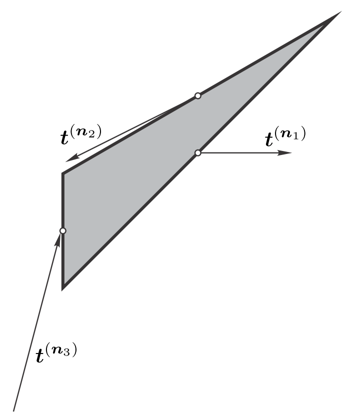
$$
(\boldsymbol{\sigma}) = \begin{pmatrix} -30.0 & -114.853 \\ -114.853 & -114.853 \end{pmatrix} \text{ MPa}.
$$

Komponente napetostnih vektorjev na drugi in tretji ravnini so

$$
\begin{aligned}
t_x^{(\boldsymbol{n}_2)} &= -30.0(-\cos 60^\circ) + (-114.853)\sin 60^\circ = -84.466 \text{ MPa}\\
t_x^{(\boldsymbol{n}_2)} &= -114.853(-\cos 60^\circ) + (-114.853)\sin 60^\circ = -42.039 \text{ MPa}\\
t_x^{(\boldsymbol{n}_3)} &= -30.0(-1.0) + (-114.853)0 = 30.0 \text{ MPa}\\
t_y^{(\boldsymbol{n}_3)} &= -114.853(-1.0) + (-114.853)0 = 114.853 \text{ MPa},
\end{aligned}
$$

torej

$$
\begin{aligned}
(\boldsymbol{t}^{(\boldsymbol{n}_2)}) &= (-84.466, -42.039) \text{ MPa}, \quad t^{(\boldsymbol{n}_2)} = 94.349 \text{ MPa},\\
(\boldsymbol{t}^{(\boldsymbol{n}_3)}) &= (30.0, 114.853) \text{ MPa}, \quad t^{(\boldsymbol{n}_3)} = 118.706 \text{ MPa}.
\end{aligned}
$$



Velikost ekstremnih normalnih napetosti izračunamo s pomočjo izraza

$$
\begin{aligned}
\sigma_{(1),(2)} &= \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}\\
&= \frac{-30.0 + (-114.853)}{2} \pm \sqrt{\left(\frac{-30.0 - (-114.853)}{2}\right)^2 + (-114.853)^2},
\end{aligned}
$$

od koder dobimo $\sigma_1 = 50.012$ MPa in $\sigma_2 = -194.865$ MPa.

Lego ravnin na kateri so glavne normalne napetosti izračunamo s pomočjo izraza

$$
\tan 2\alpha = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \frac{2(-114.853)}{-30.0 - (-114.853)} = -2.707105,
$$

od koder dobimo za nas zanimiva kota -34.863° in 55.137°. Vzemimo kot $\alpha = -34.863^\circ$ in izračunajmo kakšna normalna napetost se pojavi na ravnini določeni s kotom $\alpha = -34.863°$, tj. določeni z $(\boldsymbol{n}) = (\cos(-34.863°), \sin(-34.863°))$,

$$\begin{aligned}
\sigma_n(\alpha) &= \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\alpha + \sigma_{xy} \sin 2\alpha \\
&= \frac{-30.0 + (-114.853)}{2} + \frac{-30.0 - (-114.853)}{2} \cos\big(2(-34.863°)\big) + \\
&\quad + (-114.853) \sin\big(2(-34.863°)\big) = 50.012 \text{ MPa},
\end{aligned}$$

kar je ravno σ_1. Od tod sledi $\alpha_1 = -34.863°$, $\alpha_2 = 90° + \alpha_1 = 55.137°$. Torej na ravnini določeni z $(\boldsymbol{n}) = (\cos(-34.863°), \sin(-34.863°))$ najdemo $\sigma_1 = 50.012$ MPa, na ravnini določeni z $(\boldsymbol{n}) = (\cos 55.137°, \sin 55.137°)$ najdemo $\sigma_2 = -194.865$ MPa.

Opomba. Preverite kakšne so strižne napetosti na ravninah ekstremnih normalnih napetosti. $(\sigma_t(\alpha_1), \sigma_t(\alpha_2) \equiv 0)$

Izračunajmo še velikost in lego ekstremnih strižnih napetosti. Ekstremne strižne napetosti se pojavijo na ravninah, ki ležijo pod kotom 45°, glede na ravnine ekstremnih normalnih napetosti. Njihovo velikost izračunamo s pomočjo izraza

$$\sigma_{t,\max} = \pm\frac{\sigma_1 - \sigma_2}{2} = \pm\frac{50.012 - (-194.865)}{2} = \pm 122.434 \text{ MPa}$$

ter lego v koordinatnem sistemu x-y s pomočjo $\beta = 45° + \alpha_1 = 10.137°$. Na ravnini, določeni s kotom $\beta = 10.137°$, tj. z $(\boldsymbol{n}) = (\cos 10.137°, \sin 10.137°)$, dobimo

$$\begin{aligned}
\sigma_t(\beta) &= \frac{\sigma_{yy} - \sigma_{xx}}{2} \sin 2\beta + \sigma_{xy} \cos 2\beta \\
&= \frac{-114.853 - (-30.0)}{2} \sin(2 \cdot 10.137°) + (-114.853) \cos(2 \cdot 10.137°) \\
&= -122.434 \text{ MPa}.
\end{aligned}$$

Na pravokotni ravnini, kjer je $(\boldsymbol{n}) = (\cos 100.137°, \sin 100.137°)$, je strižna napetost

$$\begin{aligned}
\sigma_t(\beta + 90°) &= \frac{\sigma_{yy} - \sigma_{xx}}{2} \sin\big(2(\beta + 90°)\big) + \sigma_{xy} \cos\big(2(\beta + 90°)\big) \\
&= \frac{-114.853 - (-30.0)}{2} \sin(2 \cdot 100.137°) + (-114.853) \cos(2 \cdot 100.137°) \\
&= 122.434 \text{ MPa}.
\end{aligned}$$

Izračunajmo še normalno napetost na strižnih ravninah,

$$\begin{aligned}
\sigma_n(\beta) &= \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\beta + \sigma_{xy} \sin 2\beta \\
&= \frac{-30.0 + (-114.853)}{2} + \frac{-30.0 - (-114.853)}{2} \cos(2 \cdot 10.137°) + \\
&\quad + (-114.853) \sin(2 \cdot 10.137°) = -72.426 \text{ MPa}.
\end{aligned}$$

in

$$\begin{aligned}\sigma_n(\beta + 90°) &= \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos\big(2(\beta + 90°)\big) + \sigma_{xy} \sin\big(2(\beta + 90°)\big) \\ &= \frac{-30.0 + (-114.853)}{2} + \frac{-30.0 - (-114.853)}{2} \cos(2 \cdot 100.137°) + \\ &\quad + (-114.853) \sin(2 \cdot 100.137°) = -72.426 \text{ MPa}.\end{aligned}$$

Opomba. Normalno napetost na strižnih ravninah lahko izračunamo tudi neposredno, s pomočjo prve glavne invariante I_1,

$$\sigma_n(\beta) = \frac{I_1}{2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{-30.0 - 114.853}{2} = -72.426 \text{ MPa}.$$